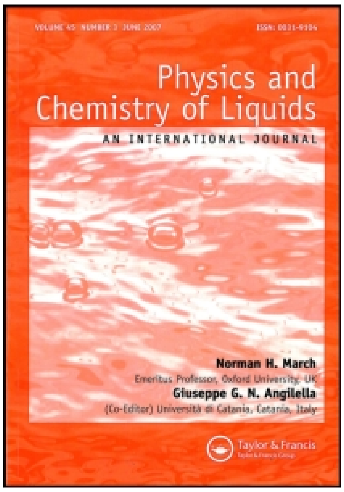


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# Gap equation for anyon superfluids

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# Gap equation for anyon superfluids

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The free energy for superconducting electrons in the Bardeen–Cooper–Schrieffer (BCS) framework is first generalized to apply to superfluid anyons. By appropriate minimization of this free energy, the gap equation for superfluid anyons is then derived. As an application, the energy gap $\Delta(0)$, in reduced energy units $k_B T_c$, with $T_c$ the transition temperature, is obtained as a function of the anyon statistics parameter.

*Keywords:* Anyon fluids and superfluids; Quantum statistics

## 1. Introduction

After the seminal works of Sutherland [1–5], Wilczek [6], and Haldane [7] (see also [8–10]), the concept of 'anyons', or particles obeying 'fractional statistics', has received considerable attention in recent years. In particular, the thermodynamics of ideal anyon gases has been extensively studied [11–16]. Although such theoretical studies have been performed with general reference to systems with arbitrary dimensionality, fractional statistics has found application especially in quasi-two-dimensional (quasi-2D) systems, such as the vortices of a strongly interacting 2D gas (2DEG) in a strong magnetic field, characterized by a fractional quantum Hall effect (FQHE) [17]. In this context, an early application was made by Lea *et al*. [18,19] to discuss the de Haas–van Alphen oscillatory orbital magnetism of a 2DEG. More recently, it has been proposed how anyon quasiparticles associated with FQHE could be possibly detected experimentally [20,21]. Moreover, Laughlin [22] has argued that the elementary excitations of Anderson's resonating valence bond model [23] might obey fractional statistics, thus supporting the idea of anyon superconductivity.

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Here, we assume indeed that quasiparticles with a gapped energy spectrum obey fractional statistics, and derive a BCS-like equation for the superconducting gap. A central quantity in such an equation is a generalized pair susceptibility, which explicitly depends on the statistical parameter $\alpha$, ranging between $\alpha = 1$ and $\alpha = 0$, corresponding to the Fermi–Dirac (FD) and Bose–Einstein (BE) limits, respectively. Our study might be of interest for quasi-2D materials, of which $MgB_2$ is among the most important examples with a superconducting critical temperature $T_c \simeq 39\,K$ [24]. We mention also the further diboride $NbB_2$ [25] which, however, has a much lower $T_c$ ($T_c = 0.62\,K$ [26]).

## 2. Gap equation for anyon superfluids

We start by considering a system of gapped quasiparticles obeying fractional statistics, with principal branch of their energy spectrum given by

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2}. \tag{1}$$



Here, $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ is the free quasiparticle dispersion relation, measured with respect to the chemical potential $\mu$, $\Delta_{\mathbf{k}}$ is the energy gap associated to pair formation, and $\mathbf{k}$ is the quasiparticle wave-vector. A microscopic derivation of equation (1) is beyond the scope of the present study. However, having in mind the diagonalization of the BCS effective Hamiltonian, such a procedure requires a suitable generalization of the Bogoliubov-Valiatin transformations for quasifermion assemblies, which in general do not obey time-reversal invariance (see [27] for a discussion). Here, we simply *assume* that such gapped quasiparticles obey anyon statistics.

Specifically, we assume that the distribution function for such superfluid anyons at inverse temperature $\beta = (k_B T)^{-1}$ is given by Wu's function [9]

$$f_\alpha(\beta E_{\mathbf{k}}) = \frac{1}{w(e^{\beta E_{\mathbf{k}}}) + \alpha}, \tag{2}$$

(see also [28–30]) where $\alpha$ denotes the statistical parameter, ranging from $\alpha = 0$ in the Bose–Einstein (BE) limit, to $\alpha = 1$ in the Fermi–Dirac (FD) limit, and $w(\zeta)$ obeys Wu's functional equation [9]

$$w^\alpha(\zeta)[1 + w(\zeta)]^{1-\alpha} = \zeta. \tag{3}$$

A sketch of Wu's distribution function, equation (2), for $0 < \alpha \leq 1$ is given in figure 1. Some properties of Wu's distribution function will be summarized in Appendix A.

The free energy for superconducting electrons (subscript $\alpha = 1$ for FD) [32]

$$F_{\alpha=1} = H_0 - \frac{1}{\beta}\sum_{\mathbf{k}}\left[\log\left(1 + e^{-\beta E_{\mathbf{k}}}\right) + \log\left(1 + e^{\beta E_{\mathbf{k}}}\right)\right], \tag{4}$$

$$H_0 = \sum_{\mathbf{k}}\left[\xi_{\mathbf{k}} + \Delta_{\mathbf{k}} b_{\mathbf{k}}^*\right], \tag{5}$$

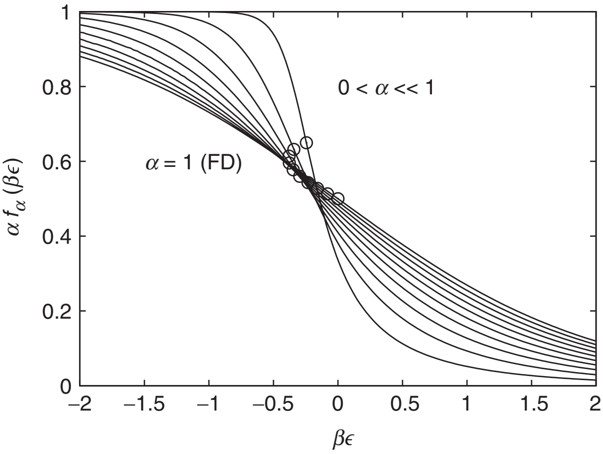


Figure 1. Distribution function for anyon statistics $f_\alpha(\beta\epsilon)$, equation (2), times statistical parameter $\alpha$, for $0 < \alpha \leq 1$, as a function of $\beta\epsilon$. Different curves refer to equally spaced values of $\alpha$ ranging between $\alpha = 0$ (BE) and $\alpha = 1$ (FD). Open circles mark the location of the inflection points, dubbed 'quasi-Fermi level' in [31].



where $b_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$ is the appropriate order parameter for the formation of singlet pairs of electrons, can be generalized straightforwardly to describe superfluid anyons as

$$F_\alpha = H_0 - \frac{1}{\beta} \sum_{\mathbf{k}} \left[ \log\left( \frac{1 + w(e^{-\beta E_{\mathbf{k}}})}{w(e^{-\beta E_{\mathbf{k}}})} \right) + \log\left( \frac{1 + w(e^{\beta E_{\mathbf{k}}})}{w(e^{\beta E_{\mathbf{k}}})} \right) \right]. \tag{6}$$

It may be checked, by direct inspection, that equation (6) reduces to equation (4) in the limit $\alpha \to 1$ (FD).

Then, minimization of the free energy, equation (6), with respect to the gap amplitude $\Delta_{\mathbf{k}}$ (i.e., $\partial F_\alpha / \partial \Delta_{\mathbf{k}} = 0$) yields the relation between the gap amplitude and the pairing order parameter:

$$b_{\mathbf{k}}^* = \Delta_{\mathbf{k}} \chi_{\mathbf{k}}^{(\alpha)}, \tag{7}$$

where $\chi_{\mathbf{k}}^{(\alpha)}$ is the generalization of the pair susceptibility to embrace arbitrary anyon statistics:

$$\chi_{\mathbf{k}}^{(\alpha)} = \frac{1}{2E_{\mathbf{k}}} [f_\alpha(-\beta E_{\mathbf{k}}) - f_\alpha(\beta E_{\mathbf{k}})] \equiv \frac{1}{2E_{\mathbf{k}}} \tanh_\alpha\left( \frac{1}{2} \beta E_{\mathbf{k}} \right). \tag{8}$$

Again, it may be checked by direct inspection that the generalized pair susceptibility $\chi_{\mathbf{k}}^{(\alpha)}$, equation (8) above, correctly reduces to the familiar expression for Fermion pairs $\chi_{\mathbf{k}}^{(\alpha=1)} \equiv \chi_{\mathbf{k}} = (2E_{\mathbf{k}})^{-1} \tanh((1/2)\beta E_{\mathbf{k}})$ in the limit $\alpha \to 1$ (FD).

Assuming a standard relation between the pairing order parameter and the gap energy, as in BCS theory [32], one eventually derives the gap equation for superfluid anyons as

$$\Delta_{\mathbf{k}} = \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}} \tanh_\alpha\left( \frac{1}{2} \beta E_{\mathbf{k}'} \right), \tag{9}$$

where $V_{\mathbf{kk}'}$ is the pairing potential. In the case of lattice models (see, e.g., [33–35]), such a potential is invariant with respect to the crystal point group, and may therefore enforce non-*s*-wave symmetries.

Summarizing, within this simple approach, the only effect of anyon statistics is encoded in the generalized pair susceptibility $\chi_{\mathbf{k}}^{(\alpha)}$, equation (8), through the 'modified' hyperbolic tangent

$$\begin{aligned} \tanh_\alpha z &\equiv f_\alpha(-2z) - f_\alpha(2z) \\ &= \frac{1}{w(e^{-2z}) + \alpha} - \frac{1}{w(e^{2z}) + \alpha}, \end{aligned} \tag{10}$$

implicitly defined in the same equation (8). Plots of the generalized pair susceptibility $\chi_{\mathbf{k}}^{(\alpha)}$ are therefore given in figure 2 as a function of $\beta E_{\mathbf{k}}$ and for several values of the statistics parameter $\alpha$. In particular, one recovers the following limiting values

$$\chi_{\mathbf{k}}^{(\alpha)} \to \frac{1}{2\alpha E_{\mathbf{k}}}, \quad \text{as } T \to 0, \tag{11a}$$

$$\to \beta \frac{w_1(1 + w_1)}{(w_1 + \alpha)^3}, \quad \text{as } E_{\mathbf{k}} \to 0, \tag{11b}$$

where $w_1 = w(1)$. From the above relations and from figure 2, one finds that $\chi_{\mathbf{k}}^{(\alpha)}$ is enhanced by a quasi-fermion value of the statistics parameter ($0 < \alpha < 1$), with respect to its behaviour in the FD limit ($\alpha = 1$).

## 3. Critical temperature and zero temperature gap

We now turn to the solution of the gap equation, equation (9), in the two limiting cases $T \to 0$ and $T \to T_c$, where the critical temperature $T_c$ is defined by the condition $\Delta_{\mathbf{k}} \to 0$. We shall limit ourselves to the case

$$V_{\mathbf{kk}'} = -\lambda \theta(\Lambda - |\xi_{\mathbf{k}}|)\theta(\Lambda - |\xi_{\mathbf{k}'}|), \tag{12}$$

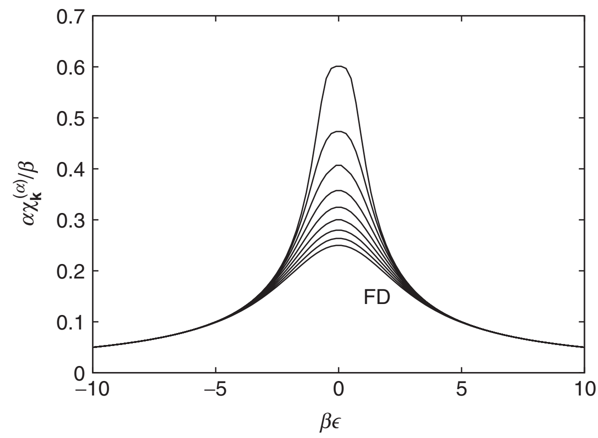


Figure 2. Generalized pair susceptibility $\alpha\chi_{\mathbf{k}}^{(\alpha)}/\beta$, equation (8), as a function of $\beta E_{\mathbf{k}}$, for different values of the statistics parameter $0 < \alpha \leq 1$, decreasing from $\alpha = 1$ (FD, bottom) to $\alpha = 0$ (BE, towards the top).

corresponding to *s*-wave symmetry, where $\Lambda > 0$ is an energy cutoff, $\lambda > 0$ the interaction strength, and $\theta(x)$ is Heaviside's step function. Higher-order symmetries can be treated with essentially the same procedure.

In the *s*-wave case, $\Delta_{\mathbf{k}} \equiv \Delta\theta(\Lambda - |\xi_{\mathbf{k}}|)$, where $\Delta = \Delta(T)$ is implicitly determined from the solution of

$$1 = -\lambda {\sum_{\mathbf{k}}}' \frac{1}{2E_{\mathbf{k}}} \tanh_\alpha\left(\frac{1}{2}\beta E_{\mathbf{k}}\right), \tag{13}$$

where the prime restricts the summation to states within the energy cutoff, $|\xi_{\mathbf{k}}| < \Lambda$.

Making use of equation (11a), in the limit $T \to 0$ one obtains

$$\Delta(0) \simeq 2\Lambda \exp\left(-\frac{\alpha}{|\lambda| N(0)}\right), \tag{14}$$

where $N(0)$ is the density of states at the energy corresponding to the inflection point in figure 1 (at a given $\alpha$), and $|\lambda| N(0) \ll 1$.

On the other hand, at $T = T_c$, making use of the approximation

$$\tanh_\alpha z \approx \begin{cases} b_\alpha z, & \text{for } |z| \leq (\alpha b_\alpha)^{-1}, \\ \alpha^{-1}, & \text{for } |z| > (\alpha b_\alpha)^{-1}, \end{cases} \tag{15}$$

where $b_\alpha = 4w_1(1 + w_1)/(w_1 + \alpha)^3$ [cf equation (11b)], and of standard BCS procedures [32], one obtains

$$k_B T_c \simeq \frac{e}{2} \alpha b_\alpha \Lambda \exp\left(-\frac{\alpha}{|\lambda| N(0)}\right). \tag{16}$$

From equations (14) and (16) it follows that

$$\frac{\Delta_0}{k_B T_c} \simeq \frac{1}{e} \frac{(w_1 + \alpha)^3}{\alpha w_1 (1 + w_1)}. \tag{17}$$

Figure 3 shows the dependence of this ratio on the statistical parameter $\alpha$.

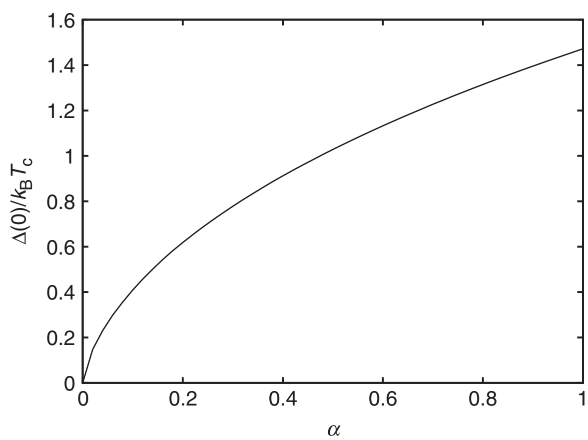


Figure 3. Ratio of energy gap $\Delta(0)$ at $T = 0$ and $k_B T_c$, equation (17), as a function of statistical parameter $\alpha$.

## 4. Conclusions

By generalization of the free energy for superconducting electrons, where Cooper pairs are bound as a result of electron–phonon interaction, we are able, by appropriate minimization, to obtain a gap equation for superfluid anyons. The potential application is to quasi-2D diborides and, in particular, $MgB_2$ and $NbB_2$. As an application of the use of the gap equation derived here, the ratio $\Delta(0)/k_B T_c$, with $\Delta(0)$ the energy gap at zero temperature, is calculated as a function of the statistical parameter $\alpha$.

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## Appendix A: Some analytical properties of Wu's distribution function for anyons

Wu's functional equation [9], equation (3), has been extensively studied in both the mathematical and the physical literature [10,11,15,16]. Explicit expressions for Wu's function $w(\zeta)$ are known for several integer and rational values of the statistical parameter $\alpha$, even outside its physical bounds $0 \leq \alpha \leq 1$. In general, Wu's function can be related to hypergeometric functions, and some useful series expansions and integral representations are known in several limits [11,16].

Here, we briefly comment on some analytical properties of Wu's distribution function $f_\alpha(z)$, equation (2), as a function of *complex* energy $z$. We start by reminding the well-known expressions of its FD ($\alpha = 1$) and BE ($\alpha = 0$) limits, which are given by

$$f_1(z) = \frac{1}{e^z + 1} \quad \text{(FD)}, \tag{18a}$$

$$f_0(z) = \frac{1}{e^z - 1} \quad \text{(BE)}. \tag{18b}$$

Both $f_1(z)$ and $f_0(z)$ are meromorphic functions, with simple poles located along the imaginary axis at $z = (2k+1)i\pi$ and $z = 2ki\pi$ ($k \in \mathbb{Z}$), i.e., at odd and even multiples of $i\pi$, respectively. These facts have direct consequences in the Mittag–Leffler expansion of the key quantity entering the definition, equation (8), of the pairing susceptibility, namely the modified hyperbolic tangent $\tanh_\alpha z$, equation (10). Such expansions for the two limiting cases of Fermions and Bosons read [36], respectively,

$$\tanh_1 z = \tanh z = 2z \sum_{k=0}^{\infty} \frac{1}{z^2 + (k + (1/2))^2 \pi^2}, \tag{19a}$$

$$-\tanh_0 z = \coth z = \frac{1}{z} + 2z \sum_{k=1}^{\infty} \frac{1}{z^2 + k^2 \pi^2}. \tag{19b}$$

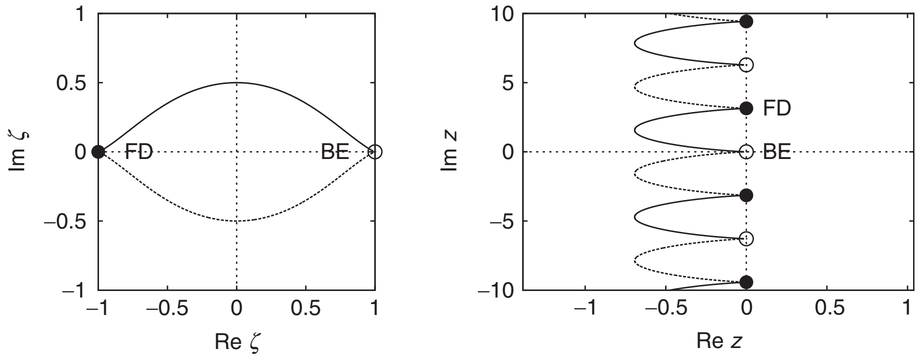


Figure 4. Left panel shows the locus of the singularities of $f_\alpha(z)$ in the complex plane $\zeta = e^z$. The solid and dashed lines refer to the plus and minus signs, respectively, in equation (20). For each value of the statistical parameter $0 < \alpha < 1$, these singularities consist of two branching points of polar type, while for $\alpha = 0$ (BE, open circle) and $\alpha = 1$ (FD, filled circle) these merge into a single pole. Right panel shows the same singularities, but now in the complex plane $z$. Because of the (purely imaginary) periodicity of the exponential function, the two singularities in the $\zeta$ variable give rise to infinitely many such singularities as a function of $z$. One recognizes the familiar poles of $f_0(z)$ (BE, open circles) and $f_1(z)$ (FD, filled circles) along the imaginary axis in the two limiting cases $\alpha = 0$ and $\alpha = 1$, respectively.



It should be noted that, while equation (19a) is used within BCS theory to compute a more accurate approximation [37] of the superconducting critical temperature, than that given in equation (16). The origin of BE condensation for assemblies with $\alpha = 0$ (Bosons) can be traced ultimately back to the presence of a single real pole ($z = 0$) in the expansion equation (19b) [37].

In the more general case of intermediate statistics ($0 < \alpha < 1$), neither expansion holds any more. By direct inspection of equation (2), one finds that Wu's distribution function $f_\alpha(z)$ is characterized by infinitely many branching points of polar type, located in the complex plane at

$$z_k^\pm = \alpha \log \alpha + (1-\alpha)\log(1-\alpha) + (2k \pm \alpha) i\pi, \tag{20}$$

where $k \in \mathbb{Z}$. Around each singularity, Wu's distribution function behaves as (cf [11])

$$f_\alpha(z) \sim \mp \frac{i}{\sqrt{2\alpha(1-\alpha)}} \frac{1}{\sqrt{z - z_k^\pm}}, \quad z \sim z_k^\pm. \tag{21}$$

The situation is depicted in figure 4. Moving away from the FD limit ($\alpha = 1$), each simple pole for $f_1(z)$ located along the imaginary axis at odd multiples of $i\pi$ [$(2k+1)i\pi$, say; filled circles in figure 4] evolve into *two* square-root-like branching points of polar type for $f_\alpha(z)$ at $z_k^+$ and $z_{k+1}^-$, respectively. The same happens to $f_\alpha(-z)$, but now with singularity pairs located in the $\mathrm{Re}\, z > 0$ half plane (not shown in figure 4).

Making use of the above discussion, and of the definition equation (10), by analogy with equation (19a), we arrive at the following conjecture for a Mittag–Leffler-like expansion of our 'modified' hyperbolic tangent:

$$\tanh_\alpha z = \frac{1}{\alpha}\sum_{k=0}^{\infty}\left(\frac{1}{\sqrt{\left(z+(1/2)z_k^+\right)\left(z+(1/2)z_{k+1}^-\right)}}-\frac{1}{\sqrt{\left(z-(1/2)z_k^+\right)\left(z-(1/2)z_{k+1}^-\right)}}\right), \tag{22}$$

where $\sqrt{z}$ denotes the principal branch of the square root function in the complex domain. Equation (22) correctly reduces to equation (19a) in the limit $\alpha \to 1$, and has been verified within numerical accuracy for $0 < \alpha \leq 1$ against equation (10) over the real axis. In addition, it fulfills the limiting behaviour equation (11a).

Moreover, by using the results of [11] and equation (10) of this article, we can express the generalized pairing susceptibility in the form of the following series:

$$\frac{1}{2z}\tanh_\alpha \frac{z}{2} = \sum_{m=0}^{\infty}(-1)^m \frac{(m\alpha+\alpha-m)_m}{m!}\frac{\sinh[(m+1)z]}{z}, \tag{23}$$

which is convergent for $0 \leq \alpha \leq 1$ and $\zeta_c < |e^z| < \zeta_c^{-1}$, with $\zeta_c = \alpha^\alpha(1-\alpha)^{1-\alpha}$ [implying $|\mathrm{Re}\, z| < -\alpha\log\alpha - (1-\alpha)\log(1-\alpha)$ along the real axis], and where $(n)_m = \Gamma(m+n)/\Gamma(n)$ denotes the Pochhammer symbol.



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